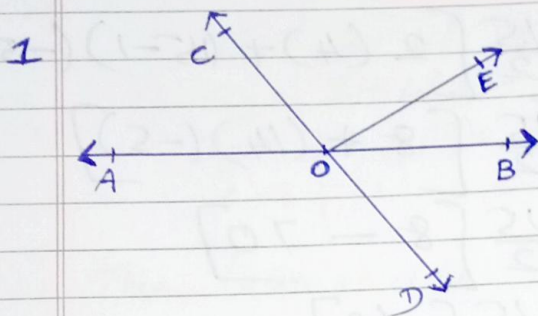


Class-9

Exercise 6.1



Given: $\angle BOD = 40^\circ$
 $\angle AOC + \angle BOE = 70^\circ$

To find: $\angle BOE$ and reflex $\angle COE$

$\angle BOD$ and $\angle AOC$ are vertically opposite angles.

Since $\angle BOD = 40^\circ \Rightarrow \angle AOC = 40^\circ$

We have $\angle AOC + \angle BOE = 70^\circ$
 $40^\circ + \angle BOE = 70^\circ$
 $\angle BOE = 70^\circ - 40^\circ$
 $\angle BOE = 30^\circ$

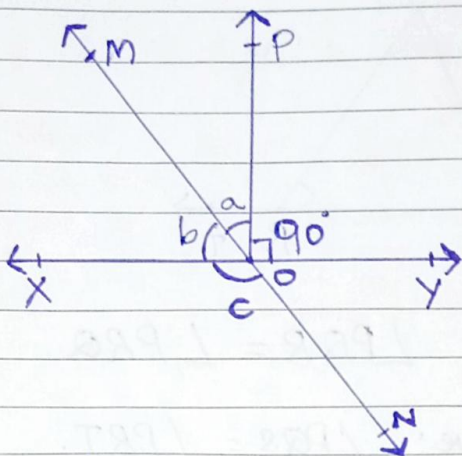
$\angle BOD$ and $\angle AOD$ are linear pair of angles.

$$\begin{aligned}\angle AOD &= 180^\circ - \angle BOD \\ &= 180^\circ - 40^\circ \\ &= 140^\circ\end{aligned}$$

Reflex angle $\angle COE = \angle AOC + \angle AOD + \angle BOD + \angle BOE$
 $= 40^\circ + 140^\circ + 40^\circ + 30^\circ$
 $= 250^\circ$

② XY and MN intersect at O.

$\angle POY = 90^\circ$ and $a:b = 2:3$ find c.



Güven : $\angle POY = 90^\circ$

$a:b = 2:3$

To find : c

$$\angle XOP + \angle POY = 180^\circ$$

$$\angle XOP = 180^\circ - \angle POY$$

$$= 180^\circ - 90^\circ$$

$$= 90^\circ$$

$$\angle XOP = \angle a + \angle b$$

$$\text{Let } a = 2x ; b = 3x$$

$$\angle XOP = 90^\circ$$

$$\Rightarrow 2x + 3x = 90^\circ$$

$$5x = 90^\circ$$

$$x = \frac{90^\circ}{5}$$

$$= 18^\circ$$

$$a = 2x = 2(18) = 36^\circ$$

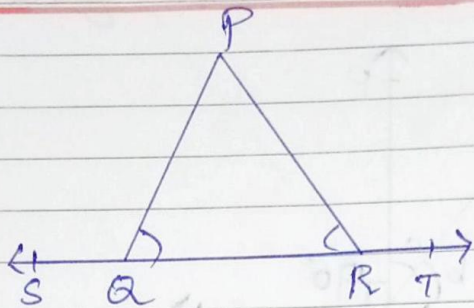
$$b = 3x = 3(18) = 54^\circ$$

$\angle b$ and $\angle NOY$ are vertically opposite.

$$\therefore \angle NOY = 54^\circ$$

$$\begin{aligned}\angle C &= \angle XON = 180^\circ - \angle NOY \\ &= 180^\circ - 54^\circ \\ &= 126^\circ\end{aligned}$$

3



Given $\angle PQR = \angle PRQ$.

To Prove: $\angle PQS = \angle PRT$.

Since ST is a straight line,

$$\angle PQR + \angle PQS = 180^\circ \rightarrow \textcircled{1}$$

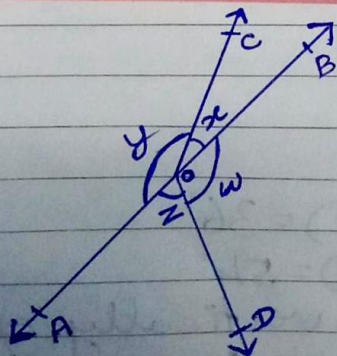
$$\angle PRQ + \angle PRT = 180^\circ \rightarrow \textcircled{2}$$

$$\underline{\angle PQR} + \underline{\angle PQS} = \underline{\angle PRQ} + \underline{\angle PRT}$$

$$\angle PQR + \angle PQS = \angle PQR + \angle PRT \quad (\text{Since } \angle PQR = \angle PRQ)$$

$$\Rightarrow \angle PQS = \angle PRT$$

Hence Proved.



Given: $x + y = w + z$
To Prove: AOB is a line.

Here $x + y + z + w$ forms a full angle.

$$x + y + w + z = 360^\circ$$

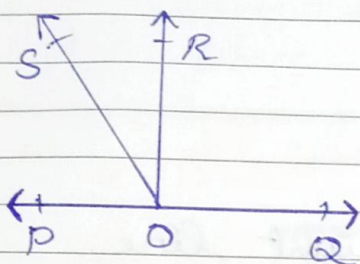
$$(x+y) + (x+y) = 360^\circ \quad (\text{Since } x+y = w+z)$$

$$2(x+y) = 360^\circ$$

$$x+y = \frac{360^\circ}{2} = 180^\circ$$

\Rightarrow AB is a straight line.

⑤



Given: POQ is a line
OR \perp OQ

To Prove: $\angle ROS = \frac{1}{2} (\angle QOS - \angle POS)$

$$POQ \text{ is a line } \Rightarrow \angle POR + \angle ROQ = 180^\circ$$

$$OR \perp OQ \Rightarrow \angle ROQ = 90^\circ$$

$$\therefore \angle POR = 180^\circ - \angle ROQ \\ = 180^\circ - 90^\circ \\ = 90^\circ$$

$$\text{Now } \angle ROS = \angle ROP - \angle POS$$

$$= 90^\circ - \angle POS \rightarrow \textcircled{1}$$

$$\text{we have } \angle ROS + \angle ROQ = \angle QOS$$

$$\Rightarrow \angle ROS = \angle QOS - \angle ROQ$$

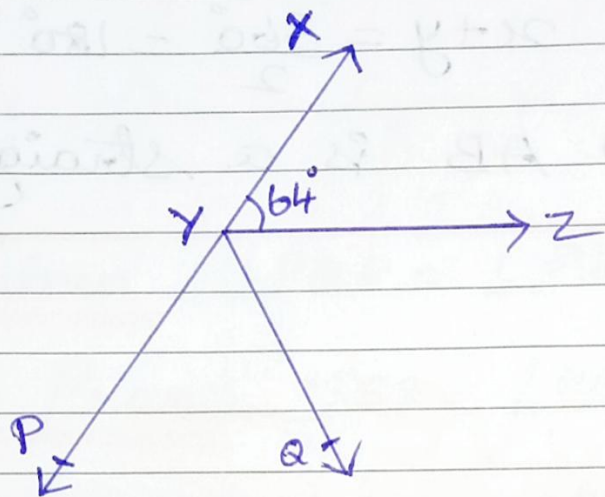
$$= \angle QOS - 90^\circ \rightarrow \textcircled{2}$$

from ① and ②, (Add ① + ②)

$$2\angle ROS = \angle QOS - \angle POS$$

$$\Rightarrow \angle ROS = \frac{1}{2} (\angle QOS - \angle POS)$$

(6)



XYP is a straight line.

$$\angle XYZ + \angle ZYQ + \angle QYP = 180^\circ$$

$$64^\circ + \angle ZYQ + \angle QYP = 180^\circ$$

$$64^\circ + 2\angle QYP = 180^\circ$$

$$\Rightarrow 2\angle QYP = 180^\circ - 64^\circ$$

(since YQ bisects $\angle XYZ$)

$$= 116^\circ$$

$$\angle QYP = \frac{116^\circ}{2} = 58^\circ$$

$$\text{Reflex } \angle QYP = 360^\circ - 58^\circ = 302^\circ$$

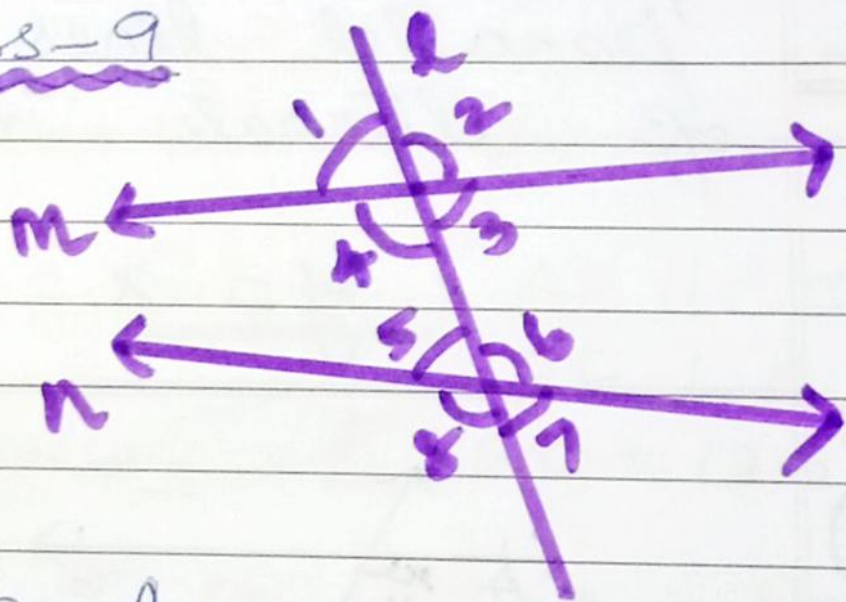
$$\text{Here } \angle XYQ = \angle XYZ + \angle ZYQ = 64^\circ + \angle QYP$$

$$\angle XYQ = 64^\circ + 58^\circ = 122^\circ$$

(since $\angle ZYQ = \angle QYP$)

$$\text{Reflex } \angle QYP = 302^\circ \text{ \& } \angle XYQ = 122^\circ.$$

class-9



Corresponding angles

$\angle 1$ and $\angle 5$

$\angle 4$ and $\angle 8$

$\angle 2$ and $\angle 6$

$\angle 3$ and $\angle 7$

Alternate interior angles

$\angle 4$ and $\angle 6$

$\angle 3$ and $\angle 5$

Alternate exterior angles

$\angle 1$ and $\angle 7$

$\angle 2$ and $\angle 8$

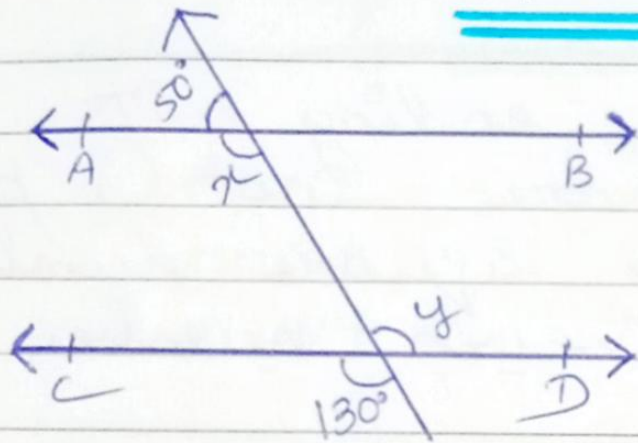
Interior angles on same side of transversal

$\angle 4$ and $\angle 5$

$\angle 3$ and $\angle 6$.

Exercise 6.2

①



To find x and y .

Here 130 and y are vertically opposite angles.
 $\therefore y = 130^\circ$

x and 50° are linear pair

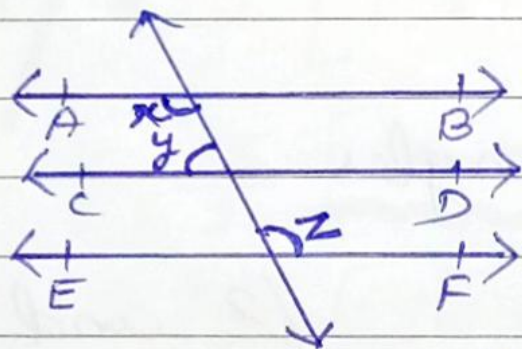
$$x + 50^\circ = 180^\circ$$

$$x = 180 - 50^\circ$$

$$= 130^\circ$$

$$\therefore x = 130^\circ, y = 130^\circ$$

②



Given : $AB \parallel CD, CD \parallel EF$
 $y : z = 3 : 7$

Since $AB \parallel CD$ and $CD \parallel EF$
 $\Rightarrow AB \parallel EF$

Thm
6.4

If a transversal intersects two parallel lines, then each pair of interior angles on same side of the transversal is supplementary.

$$x + y = 180^\circ$$

$$x = z \quad [\text{By Thm 6.2}]$$

$$z + y = 180^\circ$$

$$\text{Let } y = 3a; z = 7a$$

$$7a + 3a = 180^\circ$$

$$10a = 180^\circ$$

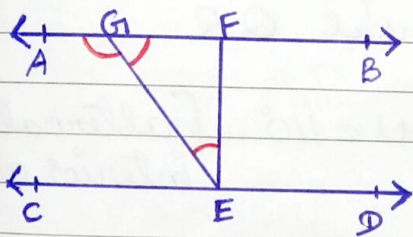
$$a = 18^\circ$$

$$z = 7(18^\circ) = 126^\circ$$

$$\Rightarrow x = z = 126^\circ$$

$$\therefore x = 126^\circ$$

3.



Given: $AB \parallel CD$
 $EF \perp CD$
 $\angle GED = 126^\circ$

To find: $\angle AGE$, $\angle GEF$,
 $\angle FGE$

$\angle AGE$ and $\angle GED$ are alternate interior angles.

$$\text{Since } \angle GED = 126^\circ \Rightarrow \underline{\angle AGE = 126^\circ}$$

$\angle AGE$ and $\angle FGE$ are linear pair of angles.

$$\angle AGE + \angle FGE = 180^\circ$$

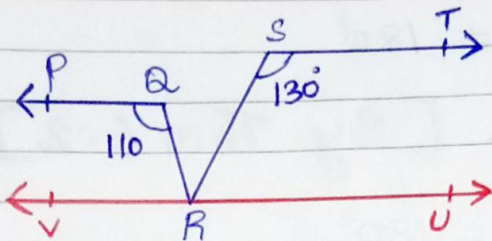
$$\angle FGE = 180^\circ - \angle AGE$$

$$\angle FGE = 180^\circ - 126^\circ = 54^\circ$$

Since $EF \perp CD \Rightarrow EF \perp AB \therefore \angle AFE = 90^\circ$

$$\underline{\angle GEF = 180^\circ - \angle FGE - 90^\circ = 180^\circ - 54^\circ - 90^\circ = 36^\circ}$$

④



Güven: $PQ \parallel ST$
 $\angle PQR = 110^\circ$
 $\angle RST = 130^\circ$

To Find: $\angle QRS$

Construction: Draw $RU \parallel ST$

We have $PQ \parallel ST$, $ST \parallel RU$
 $\Rightarrow PQ \parallel RU$

PQ and RU are parallel lines cut by transversal QR .

$$\angle PQR = \angle QRU = 110^\circ \quad [\text{alternate interior angles}]$$

ST and RU are parallel lines cut by transversal SR .

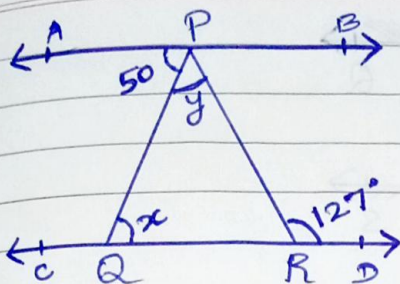
$$\angle RST + \angle SRU = 180^\circ$$

$$130^\circ + \angle SRU = 180^\circ$$

$$\angle SRU = 180^\circ - 130^\circ = 50^\circ$$

$$\begin{aligned} \angle QRS &= \angle QRU - \angle SRU \\ &= 110^\circ - 50^\circ \\ &= 60^\circ \end{aligned}$$

$$\therefore \angle QRS = 60^\circ$$



Given: $AB \parallel CD$

$$\angle APQ = 50^\circ$$

$$\angle PRD = 127^\circ$$

To Find: x and y

AB and CD are parallel lines cut by the transversal PQ

$\angle APQ$ and x are alternate interior angles.

$$\therefore x = \angle APQ = 50^\circ$$

$$x = 50^\circ$$

AB and CD are parallel lines cut by the transversal PR .

$\angle APR$ and $\angle PRD$ are alternate interior angles.

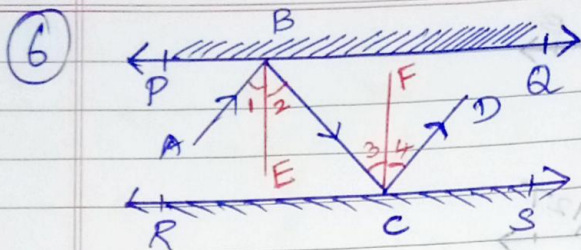
$$\therefore \angle APR = \angle PRD = 127^\circ$$

$$50^\circ + y = 127^\circ$$

$$y = 127^\circ - 50^\circ$$

$$= 77^\circ$$

$$x = 50^\circ ; y = 77^\circ$$



Construction : Draw $BE \perp PQ$
 $CF \perp RS$

As $PQ \parallel RS$
 $\therefore BE \parallel CF$

BE and CF are two parallel lines cut by the transversal BC at B and C .

$\angle 2 = \angle 3$ (alternate interior angles)

Also $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$
 (angle of incidence = angle of reflection)

Also $\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\angle ABC = \angle DCB$
 $\Rightarrow AB \parallel CD$

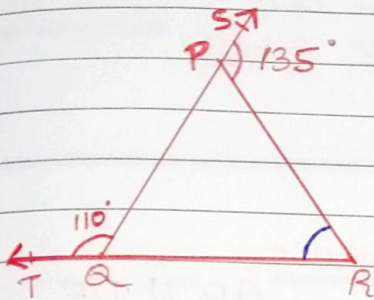
Angle Sum Property of a Triangle

Theorem: The sum of the angles of a triangle is 180° .

Theorem: If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Exercise 6.3

①



Given:- $\angle SPR = 135^\circ$
 $\angle PQT = 110^\circ$

To find: $\angle PRQ$

$$\angle PQT + \angle PQR = 180^\circ \quad (\text{linear pairs})$$

$$110^\circ + \angle PQR = 180^\circ$$

$$\Rightarrow \angle PQR = 180^\circ - 110^\circ$$

$$= 70^\circ$$

$$\angle PQR = 70^\circ$$

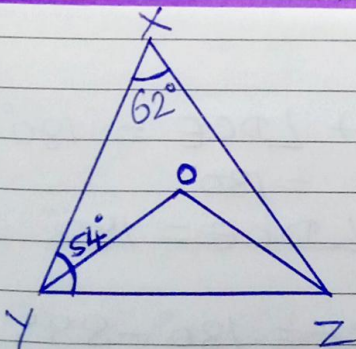
$$\angle SPR = \angle PQR + \angle PRQ$$

$$135^\circ = 70^\circ + \angle PRQ$$

$$135^\circ - 70^\circ = \angle PRQ$$

$$\therefore \angle PRQ = 65^\circ$$

②



In $\triangle XYZ$, By Angle sum property $\angle X + \angle Y + \angle Z = 180^\circ$

$$62^\circ + 54^\circ + \angle Z = 180^\circ$$

$$116^\circ + \angle Z = 180^\circ$$

$$\angle Z = 180^\circ - 116^\circ = 64^\circ$$

$$\angle OYZ = \frac{54^\circ}{2} = 27^\circ \quad (\text{Since } OY \text{ bisects})$$

$$\angle OZY = \frac{64^\circ}{2} = 32^\circ \quad (\text{Since } OZ \text{ bisects})$$

By angle sum property

$$\angle O + \angle OZY + \angle OYZ = 180^\circ$$

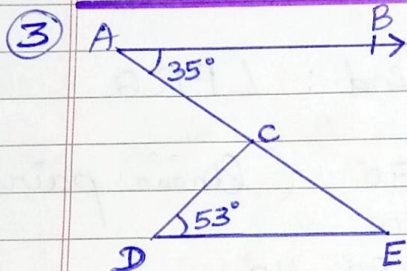
$$\angle O + 32^\circ + 27^\circ = 180^\circ$$

$$\angle O + 59^\circ = 180^\circ$$

$$\angle O = 180^\circ - 59^\circ$$

$$= 121^\circ$$

$$\angle O = \angle YOZ = 121^\circ$$



Given:-

$AB \parallel DE$

$\angle BAC = 35^\circ$

$\angle CDE = 53^\circ$

AB and DE are parallel lines cut by the transversal AE .

$$\angle BAC = \angle CED \text{ (alternate angles)}$$

$$\Rightarrow \angle CED = 35^\circ$$

In $\triangle CDE$,

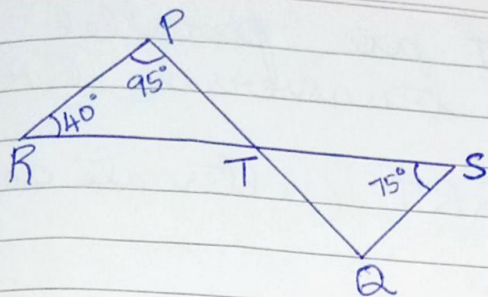
$$\angle CDE + \angle CED + \angle DCE = 180^\circ$$

$$53^\circ + 35^\circ + \angle DCE = 180^\circ$$

$$88^\circ + \angle DCE = 180^\circ$$

$$\angle DCE = 180^\circ - 88^\circ$$

$$= 92^\circ$$



Given: PQ and RS intersect at T .
 $\angle PRT = 40^\circ$
 $\angle RPT = 95^\circ$
 $\angle TSQ = 75^\circ$

To find: $\angle SQT$.

In $\triangle RPT$, $\angle P + \angle R + \angle T = 180^\circ$

$$95^\circ + 40^\circ + \angle T = 180^\circ$$

$$135^\circ + \angle T = 180^\circ$$

$$\angle T = 180^\circ - 135^\circ$$

$$\angle PTR = \angle T = 45^\circ$$

$\angle PTR$ and $\angle STQ$ are vertically opposite angles.

$$\therefore \angle STQ = 45^\circ$$

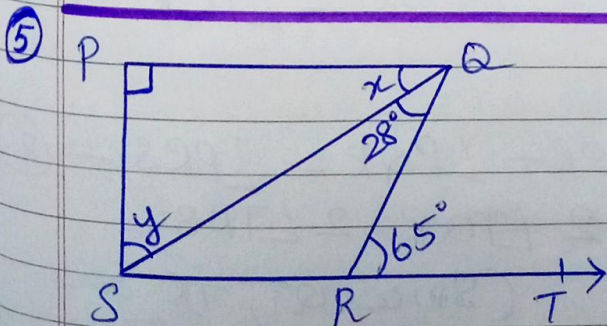
In $\triangle STQ$, $\angle STQ + \angle TSQ + \angle SQT = 180^\circ$

$$45^\circ + 75^\circ + \angle SQT = 180^\circ$$

$$120^\circ + \angle SQT = 180^\circ$$

$$\angle SQT = 180^\circ - 120^\circ$$

$$= 60^\circ$$



Given:- $PQ \parallel RS$
 $PQ \perp PS$

$$\angle SQR = 28^\circ$$

$$\angle QRT = 65^\circ$$

To find: x and y

PQ and ST are parallel lines cut by the transversal QR

$$\angle PQR = \angle QRT \text{ (alternate angles)}$$

$$x + 28^\circ = 65^\circ$$

$$x = 65^\circ - 28^\circ$$

$$= 37^\circ$$

$$\angle P + x + y = 180^\circ$$

$$\angle P = 90^\circ, \angle x = 37^\circ$$

$$\therefore 90^\circ + 37^\circ + y = 180^\circ$$

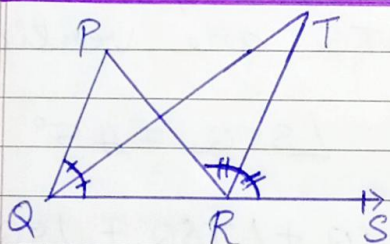
$$127^\circ + y = 180^\circ$$

$$y = 180^\circ - 127^\circ$$

$$= 53^\circ$$

$$\angle x = 37^\circ ; \angle y = 53^\circ$$

(6)



To prove: $\angle QTR = \frac{1}{2} \angle QPR$

In $\triangle QRT$,

$$\angle QTR + \angle TQR = \angle TRS$$

$$\Rightarrow \angle QTR = \angle TRS - \angle TQR \rightarrow (1)$$

In $\triangle PQR$

$$\angle QPR + \angle PQR = \angle PRS \rightarrow (2)$$

$$\angle QPR + 2 \angle TQR = 2 \angle TRS$$

(Since QT, TR are angle bisectors)

$$\begin{aligned}\angle QPR &= 2 \angle TRS - 2 \angle TQR \\ &= 2 (\angle TRS - \angle TQR)\end{aligned}$$

$$\angle QPR = 2 \angle QTR \text{ (using (1))}$$

$$\angle QTR = \frac{1}{2} \angle QPR$$

Hence Proved.