CHAPTER 2 - COMPLEX NUMBERS.

Why complex numbers?

Consider $x^2+1=0$

$$x^2=-1$$

$$x=\pm\sqrt{-1}$$

It has no solution. When we square a real number it is impossible to get a negative real number.

Let us denote $\sqrt{-1}$ as i.

$i^2=-1$, $i \to$ imaginary unit.

$i^0=1$; $i^1=i$; $i^2=-1$; $i^3=i^2\cdot i=-1\cdot i=-i$

$i^4=i^2\cdot i^2=-1\cdot(-1)=1$

$$i^{-1}=\frac{1}{i}=\frac{i}{i^2}=\frac{i}{-1}=-i \;;\; i^{-2}=\frac{1}{i^2}=\frac{1}{-1}=-1$$

$$i^{-3}=\frac{1}{i^3}=\frac{1}{i^2\cdot i}=\frac{1}{-1\cdot i}=\frac{-1}{i}=-(-i)=i$$

$$i^{-4}=\frac{1}{i^4}=\frac{1}{1}=1$$

Note $\boxed{(i)^{-4}=i^4}$

For any integer n,

$\boxed{n=4q+k}$, $0\le k<4$.

where k, q are integers.

$$(i)^n=(i)^{4q+k}=i^{4q}\cdot i^k=(i^4)^q\cdot i^k=1\cdot i^k=i^k$$

Exercise 2.1

Simplify the following.

1. $$i^{1947}+i^{1950} = i^{1944}\cdot i^{3} + i^{1948}\cdot i^{2}$$
$$= i^{3}+i^{2}$$
$$= -i-1 = -1-i$$

2. $$i^{1948} - i^{-1869} = i^{1948} - \frac{1}{i^{1869}}$$
$$= 1-\frac{1}{i} = 1-(-i) = 1+i$$

3. $$\sum_{n=1}^{12} i^{n} = i+i^{2}+i^{3}+i^{4}+i^{5}+i^{6}+i^{7}+i^{8}+i^{9}+i^{10}+i^{11}+$$
$$=(i-1-i+1)+(i-1-i-1)+(i-1-i+$$
$$= 0+0+0=0$$

4. $$i^{59}+\frac{1}{i^{59}} = i^{3}+\frac{1}{i^{3}} = -i+i=0$$

5. $$i\cdot i^{2}\cdot i^{3}\cdots i^{2000} = i^{1+2+3+\cdots+2000}$$
$$= i^{\frac{2000\times 2001}{2}}$$
$$= i^{1000\times 2001}$$
$$= i^{4\times 250\times 2001}$$
$$= i^{4k} \quad \text{where } k = 250\times 20$$
$$= 1. \quad (\text{Since } i^{4n}=1)$$

6. $$\sum_{n=1}^{10} i^{n+50} = i^{51}+i^{52}+i^{53}+\cdots+i^{60}$$
$$= i^{50}(i+i^{2}+i^{3}+\cdots+i^{10})$$
$$= i^{2}(0+0+\cdots+i^{9}+i^{10})$$
$$= -1\,(0+0+\cdots+i+i^{2})$$
$$= -1(i-1)$$
$$= 1-i$$