Rectangular form of complex numbers.

$x+iy \qquad x, y \in R.$

Real part Imaginary part.

Let $Z_1 = x_1 + iy_1$; $Z_2 = x_2 + iy_2$

$Z_1 = Z_2 \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$.

i) If $Z = x+iy$ and $k \in R$ then
$kZ = kx + i(ky)$.

ii) If $Z_1 = x_1 + iy_1$ and $Z_2 = x_2 + iy_2$

then $Z_1 + Z_2 = (x_1 + x_2) + i(y_1 + y_2)$

$Z_1 - Z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$$\begin{aligned} Z_1 Z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1 x_2 + i x_1 y_2 + i y_1 x_2 - y_1 y_2 \\ &= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1). \end{aligned}$$

iii) If $Z = x+iy$ then
$iZ = i(x+iy)$
$= -y + ix$

Exercise 2.2

1. Evaluate the following if $z = 5-2i$ and $w = -1+3i$

(i) $z + w = 5 - 2i - 1 + 3i = 4 + i$

(ii) $z - iw$.

$z = 5 - 2i$

$w = -1 + 3i$

$iw = -i + 3i^2$

$-iw = i - 3i^2 = i - 3(-1) = i + 3$.

$z - iw = 5 - 2i + i + 3 = 8 - i$

(iii) $2z + 3w$

$z = 5 - 2i$, $w = -1 + 3i$

$2z = 10 - 4i$, $3w = -3 + 9i$

$2z + 3w = 10 - 4i - 3 + 9i$

$= 7 + 5i$

(iv) $zw = (5 - 2i)(-1 + 3i)$

$= -5 + 15i + 2i - 6i^2$

$= -5 + 17i - 6(-1)$ $(\because i^2 = -1)$

$= -5 + 17i + 6$

$= 1 + 17i$

(v) $z^2 + 2zw + w^2 = (5 - 2i)^2 + 2(5 - 2i)(-1 + 3i) + (-1 + 3i)^2$

$= 25 - 20i + (2i)^2 + 2[-5 + 15i + 2i + 6] + [1 - 6i + (3i)^2]$

$= 25 - 20i - 4 + 2[1 + 17i] + [1 - 6i - 9]$

$= 25 - 20i - 4 + 2[1 + 17i] + [-8 - 6i]$

$= 25 - 20i - 4 + 2 + 34i - 8 - 6i$

$= 15 + 8i$

(vi) $(z + w)^2 = (5 - 2i - 1 + 3i)^2$

$= (4 + i)^2$

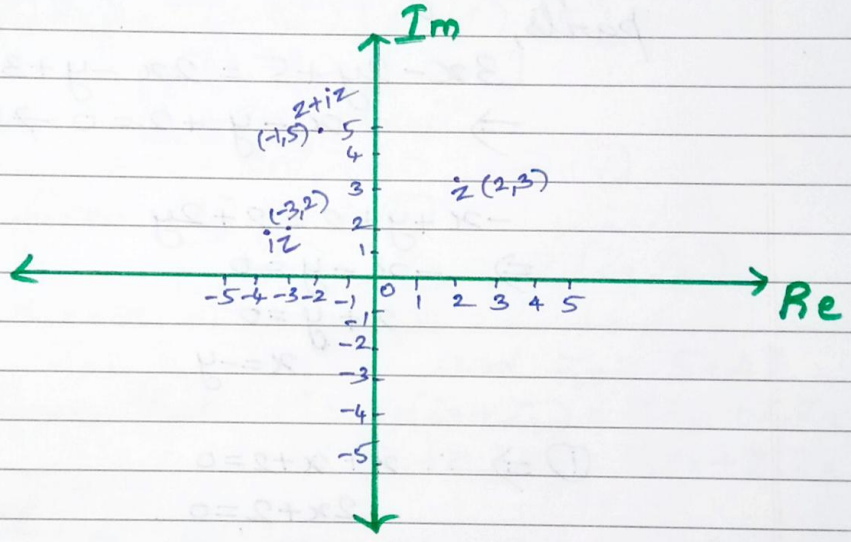
$= 16 + 8i + i^2 = 16 + 8i - 1$ $(\because i^2 = -1)$

$= 15 + 8i$

2. Given the complex number $z = 2+3i$, represent the complex number on Argand plane.

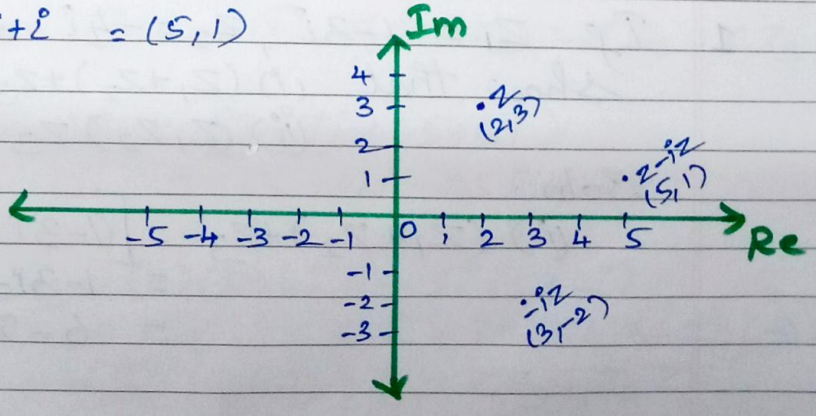
(i) $z, iz, z+iz$

$z = 2+3i = (2,3)$
$iz = 2i-3 = (-3,2)$
$z+iz = -1+5i = (-1,5)$



(ii) $z, -iz, z-iz$

$z = 2+3i = (2,3)$
$iz = 2i-3 = (-3,2)$
$-iz = 3-2i$
$z-iz = 5+i = (5,1)$



3. Find the values of the real numbers x and y if $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal.

$$(3-i)x-(2-i)y+2i+5 = 2x+(-1+2i)y+3+2i$$
$$3x-ix-2y+iy+2i+5 = 2x-y+2iy+3+2i$$
$$(3x-2y+5)+i(2+y-x) = (2x-y+3)+i(2+2y)$$

Equating the real and imaginary parts,

$$3x-2y+5 = 2x-y+3$$
$$\Rightarrow \quad x-y+2=0 \rightarrow (1)$$

$$-x+y+2 = 2+2y$$
$$\Rightarrow \quad -x-y=0$$
$$x+y=0$$
$$x=-y$$

$$(1) \Rightarrow \therefore \quad x+x+2=0$$
$$2x+2=0$$
$$2x=-2$$
$$x=-1$$
$$y=1$$