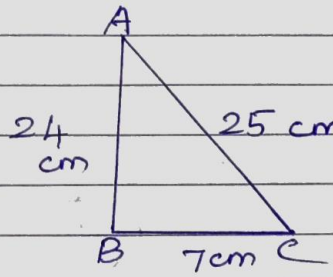
Class-10
Exercise 8.1

1. In $\triangle ABC$, right-angled at B, AB = 24cm, BC = 7cm. Determine

(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$



Soln:

i) $\sin A = \frac{\text{opp side}}{\text{hypotenuse}}$

$= \frac{7}{25}$

$AC = \sqrt{24^2+7^2}$

$= \sqrt{576+49}$

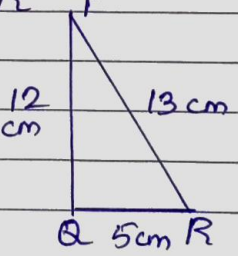
$= \sqrt{625}$

$= 25$

$\cos A = \frac{24}{25}$

ii) $\sin C = \frac{24}{25}$ $\cos C = \frac{7}{25}$

2. In Fig. find $\tan P - \cot R$



In $\triangle PQR$,

$PQ^2 + QR^2 = PR^2$

$QR^2 = PR^2 - PQ^2$

$= 13^2 - 12^2$

$= 169 - 144$

$= 25$

$QR = \sqrt{25} = 5$

$\tan P = \frac{\text{opp. side}}{\text{adj side}}$ $\cot R = \frac{\text{adj side}}{\text{opp. side}}$

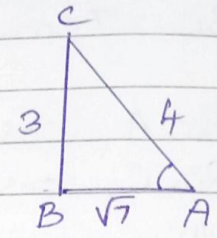
$= \frac{5}{12}$ $= \frac{5}{12}$

$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0.$

3. If $\sin A = \frac{3}{4}$ calculate $\cos A$ and $\tan A$.

Soln:

$$\sin A = \frac{3}{4}$$



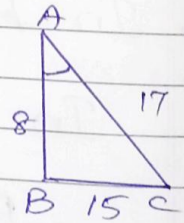
$$\cos A = \frac{adj}{hyp} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{opp}{adj} = \frac{3}{\sqrt{7}}$$

$$AC^2 = AB^2 + BC^2$$
$$4^2 = AB^2 + 9$$
$$16 = AB^2 + 9$$
$$AB^2 = 16 - 9$$
$$= 7$$
$$AB = \sqrt{7}$$

4. Given $15 \cot A = 8$, find $\sin A$ and $\sec A$

$$15 \cot A = 8$$
$$\cot A = \frac{8}{15} = \frac{adj}{opp.}$$



$$\sin A = \frac{opp}{hyp} = \frac{15}{17}$$

$$15^2 + 8^2 = AC^2$$
$$225 + 64 = 289$$
$$AC = \sqrt{289} = 17$$

$$\sec A = \frac{hyp}{adj} = \frac{17}{8}$$

5. Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

$$\sec \theta = \frac{hyp}{adj} = \frac{13}{12}$$

$AB^2 + BC^2 = AC^2$

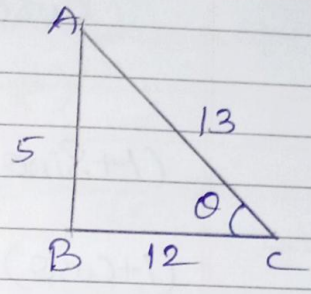
$AB^2 + (12)^2 = (13)^2$

$AB^2 = (13)^2 - (12)^2$

$= 169 - 144$

$= 25$

$AB = \sqrt{25} = 5$



$\sin\theta = \frac{5}{13}$ $\quad \tan\theta = \frac{5}{12}$ $\quad \operatorname{cosec}\theta = \frac{13}{5}$

$\cos\theta = \frac{12}{13}$ $\quad \cot\theta = \frac{12}{5}$ $\quad \sec\theta = \frac{13}{12}$

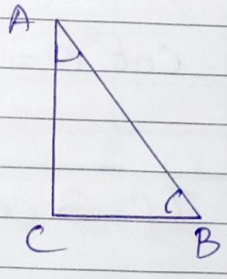
6. If $\angle A$ and $\angle B$ are acute angle such that $\cos A = \cos B$ then show that $\angle A = \angle B$.

Given $\cos A = \cos B$

$\frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow AC = BC$

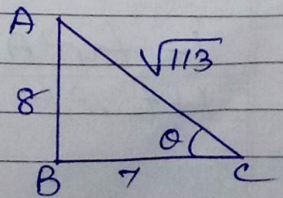
$\Rightarrow \angle A = \angle B$ [using the theorem sides opposite to equal angles of a triangle are equal]



7. If $\cot\theta = \frac{7}{8}$, evaluate (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$\cot\theta = \frac{7}{8} = \frac{adj}{opp.}$

$\sin\theta = \frac{opp}{hyp} = \frac{8}{\sqrt{113}}$



$$\cos\theta = \frac{7}{\sqrt{113}}$$

$$(1+\sin\theta)(1-\sin\theta) = \left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)$$

$$(1+\cos\theta)(1-\cos\theta) \quad \left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$.

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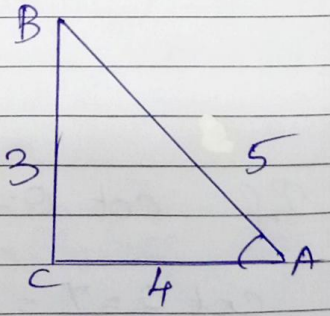
8. If $3\cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.



$3\cot A = 4$

$\cot A = \frac{4}{3}$

$\Rightarrow \tan A = \frac{3}{4}$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$$\cos^2 A - \sin^2 A = (\cos A)^2 - (\sin A)^2$$
$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}.$$

$$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$$

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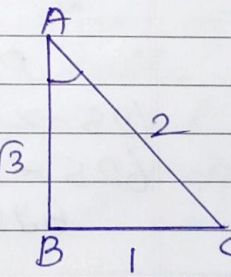
9. In triangle ABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$ find the value of

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$



Given $\tan A = \frac{1}{\sqrt{3}} = \frac{\text{opp}}{\text{adj}}$

$AC = \sqrt{3+1} = 2$

$\sin A = \frac{1}{2}$; $\cos A = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\sqrt{3}}{2}$; $\cos C = \frac{1}{2}$

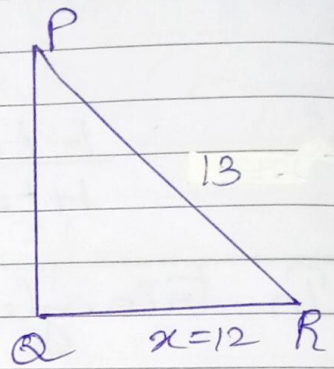
i) $\sin A \cos C + \cos A \sin C = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{4} + \frac{3}{4} = 1$

ii) $\cos A \cos C - \sin A \sin C = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= 0$

10) In ΔPQR right-angled at Q, $PR+QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.



Soln:-

Given: $PQ = 5$ cm

$PR + QR = 25$ cm

Let $QR = x$

$\therefore PR = 25 - x$.

By Pythagoras theorem.

$$PR^2 = PQ^2 + QR^2$$

$$(25-x)^2 = 5^2 + x^2$$

$$625 - 50x + x^2 = 25 + x^2$$

$$625 - 25 - 50x + x^2 - x^2 = 0$$

$$600 - 50x = 0$$

$$600 = 50x$$

$$\frac{600}{50} = x$$

$$x = 12 \text{ cm}$$

$\Rightarrow QR = 12$ cm

$PR = 25 - 12 = 13$.

$\sin P = \frac{12}{13}$; $\cos P = \frac{5}{13}$; $\tan P = \frac{12}{5}$

ii) i) False
ii) True
iii) False
iv) False
v) False