Exercise 2.3

1. If $Z_1 = 1-3i$, $Z_2 = -4i$ and $Z_3 = 5$ show that (i) $(Z_1+Z_2)+Z_3 = Z_1+(Z_2+Z_3)$

(ii) $(Z_1 Z_2) Z_3 = Z_1 (Z_2 Z_3)$

Soln:

(i) $(Z_1 + Z_2) + Z_3 = [(1-3i) - 4i] + 5$

$= 1-3i-4i+5$

$= 6-7i \longrightarrow (1)$

$$z_1 + (z_2 + z_3) = 1 - 3i + (-4i + 5)$$
$$= 1 - 3i - 4i + 5$$
$$= 6 - 7i \rightarrow (2)$$

from (1) and (2) $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

ii) $(z_1 z_2) z_3 = [(1-3i)(-4i)](5)$
$$= [-4i + 12i^2](5)$$
$$= (-4i - 12)5 = -20i - 60 \rightarrow (3)$$

$$z_1(z_2 z_3) = (1-3i)[(-4i)(5)]$$
$$= (1-3i)[-20i]$$
$$= -20i + 60i^2$$
$$= -20i - 60 \rightarrow (4)$$

from (3) and (4)
$$(z_1 z_2) z_3 = z_1(z_2 z_3)$$

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2. If $z_1 = 3$, $z_2 = -7i$ and $z_3 = 5 + 4i$
Show that (i) $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$
(ii) $(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$.

Proof:
$$z_1 = 3, \quad z_2 = -7i, \quad z_3 = 5 + 4i$$

(i) $z_1(z_2 + z_3) = 3(-7i + 5 + 4i)$
$$= -21i + 15 + 12i = 15 - 9i \rightarrow (1)$$

$$z_1 z_2 + z_1 z_3 = 3(-7i) + (3)(5 + 4i)$$
$$= -21i + 15 + 12i = 15 - 9i \rightarrow (2)$$

from (1) and (2) $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$.

ii) $(z_1 + z_2) z_3 = (3 - 7i)(5 + 4i)$
$$= 15 + 12i - 35i + 28$$
$$= 43 - 23i \rightarrow (1)$$

$$z_1 z_3 + z_2 z_3 = (3)(5 + 4i) - 7i(5 + 4i)$$
$$= 15 + 12i - 35i + 28 = 43 - 23i \rightarrow (2)$$

from (1) and (2) $(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$

3. If $z_1 = 2+5i$, $z_2 = -3-4i$ and $z_3 = 1+i$ find the additive and multiplicative inverse of z_1, z_2 and z_3.

Given $z_1 = 2+5i$
$z_2 = -3-4i$
$z_3 = 1+i$

Additive inverse:
$-z_1 = -2-5i$
$-z_2 = 3+4i$
$-z_3 = -1-i$

Multiplicative inverse.

Let $z = x+iy$.

$zz^{-1} = 1$

$$\Rightarrow z^{-1} = \frac{1}{z} = \frac{1}{x+iy} = \frac{1}{x+iy} \times \frac{x-iy}{x-iy}$$

$$= \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}.$$

$$z_1^{-1} = \frac{2}{2^2+5^2} + i\frac{(-5)}{2^2+5^2} = \frac{2}{29} - \frac{5i}{29}$$

$$= \frac{1}{29}(2-5i)$$

$$z_2^{-1} = \frac{(-3)}{(-3)^2+(-4)^2} + i\left(\frac{4}{(-3)^2+(-4)^2}\right) = \frac{-3}{9+16} + i\frac{4}{9+16}$$

$$= \frac{-3}{25} + \frac{4i}{25}$$

$$= \frac{1}{25}(-3+4i)$$

$$z_3^{-1} = \frac{1}{1^2+1^2} + i\frac{(-1)}{1^2+1^2} = \frac{1}{2} - \frac{i}{2}$$

$$= \frac{1}{2}(1-i).$$